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NUMERICAL ANALYSIS OF PROBABILITY CHARACTERISTICS FOR TEMPERATURE PROCESSES OF FLUIDIZED-BED TREATMENT OF MATERIAL

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Probability characteristics of the duration of stay, temperature, and degree of burning for particles in a fluidized bed are investigated. An unknown-boundary problem is used as a model to calculate the process of limestone particle dissociation.

Introduction. The random nature of the duration of particle stay in the working zone due to random motions and collisions of the particles is a special feature of the technological processes of fluidized-bed treatment of material. The time of material treatment is one of the most important technological parameters that governs the degree of completeness for a technological process and the capacity. Since the duration of particle stay is the random quantity, particle characteristics, such as temperature and the degree of burning, turn out to be the random quantities, too. To develop the theoretical-estimation methods for the degree of completeness of the physicochemical processes in question, we need to have the adequate mathematical description of particle characteristics that must be performed in probability-theoretic terms.

Mathematical Model. For a theoretical estimation of the probability characteristics of the duration of particle stay in a fluidized bed, use is made of a computational experiment on calculating the variation in the concentration of tagged particles at the outlet from the working zone [1]. The process is modeled on the basis of a mathematical model of the diffusion process of the particles and transfer motion of a bed with a specified flow velocity. As a result of the computational experiment we are able to obtain complete information on the probability characteristics of the duration of particle stay. The magnitudes of the initial and central moments can also be calculated by the final algebraic expressions that are derived analytically in terms of the coefficients of the mathematical model of the diffusion process [1]. The expectation of the duration of particle stay coincides with the calculated time of the regime of ideal displacement $m_\tau = L/u$. The variance of the duration of particle stay in the bed is calculated by the following relation [1]:

$$D_\tau = m_\tau^2 \frac{2}{\mathrm{Pe}} \left[1 + \frac{1}{\mathrm{Pe}} (\exp(-\mathrm{Pe}) - 1) \right] . \tag{1}$$

As a mathematical model for calculating the process of the heating of a spherically shaped particle and its subsequent burning accompanied by the reaction of limestone dissociation, let us use a problem with the unknown boundary [2] that separates the reacted portion of particle material from the unreacted portion (Stefan type problem):

$$\rho_i c_i \frac{\partial T_i(x, t)}{\partial t} = \lambda_i \left(\frac{\partial T_i^2(x, t)}{\partial x^2} + \frac{2}{x} \frac{\partial T_i(x, t)}{\partial x} \right) \tag{2}$$

for $i = 1$, $x \in [0; s(t)]$; $i = 2$, $x \in [s(t); R]$;

$$s(0) = s_0, \ T_1(x, 0) = T_0^1(x), \ x \in [0; s_0], \ T_2(x, 0) = T_0^2(x), \ x \in [s_0, R]; \tag{3}$$

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$$\frac{\partial T_1(0, t)}{\partial x} = 0 ; \quad (4)$$

$$\lambda_2 \frac{\partial T_2(R, t)}{\partial x} = \alpha^{\Sigma} [T_{\text{h.m}} - T(R, t)] ; \quad (5)$$

$$T_i(s(t), t) = T^* = \text{const}, \quad i = 1, 2 ; \quad (6)$$

$$\lambda_1 \frac{\partial T_1(s, t)}{\partial x} - \lambda_2 \frac{\partial T_2(s, t)}{\partial x} = \kappa \rho_1 1 (T - T^*) \frac{ds}{dt} ; \quad (7)$$

where (2) are the one-dimensional heat conduction equations for limestone ($i = 1$) and lime ($i = 2$); (3) are the initial conditions for the position of the boundary between the lime and limestone and for the temperature distributions; (4) and (5) are the boundary conditions at the center of the particle and on its surface; (6) is the continuity condition for the temperature field; (7) is the equation for the unknown boundary (the Stefan condition).

To calculate mathematical model (2)-(7), we employed a finite-difference approximation of the boundary problem with explicit separation of the position coordinate for the unknown boundary [3]. In the calculations, we used the following thermophysical characteristics: the latent heat of the reaction of limestone dissociation $\kappa = 1640$ kJ; the limestone density $\rho_1 = 2600$ kg/m^3; the lime density $\rho_2 = 1600$ kg/m^3; the temperature of the dissociation reaction $T^* = 850^{\circ}$C; the particle radius $R = 0.024$ m; the total heat-transfer coefficient $\alpha^{\Sigma} = 250$ W/(m$^2\cdot{}^{\circ}$C).

Statement of the Problem of Analysis and Its Solution Method. The problem of analysis of the probability characteristics for a temperature process of particle burning consists in determination of the distribution law, expectations, variances, central and initial moments of a higher order from the specified probability characteristics of the duration of stay.

Exhaustive information on a random quantity is carried by the differential distribution of the investigated parameter. Calculation of the distribution laws is a complicated problem while information on several initial and central moments of the random quantity will suffice for both the analysis and practical conclusions. The complexity of the general approaches of a probability-theoretic investigation of nonlinear dynamic processes makes necessary approximate methods for the analysis of statistical dynamics [4]. One of the most economical and accurate methods is the interpolation method, which is similar to the Monte Carlo method in versatility. As far as computation is concerned, the interpolation method is the successive employment of Gauss quadrature formulas, where the probability density distribution for the random quantity is used as the weighting functions [4].

In accordance with the interpolation method, the expectation and variance of the quantity in question (for example, the degree of burning) are calculated by the following formulas:

$$M[\gamma(\tau)] \approx \sum_{i=1}^{n} r_i \gamma(m_\tau + \sigma_\tau \nu_i),$$

$$D[\gamma(\tau)] \approx \sum_{i=1}^{n} r_i [\gamma(m_\tau + \sigma_\tau \nu_i) - m_\gamma]^2 .$$

The Chebyshev nodes and Christoffel numbers that are required for the calculations are calculated for random quantities with a uniform, normal, or exponential law of probability distribution [4]. The calculations of probability characteristics by the interpolation method with different powers of the approximating polynomials showed that, when $n = 5$ and $n = 7$, the results of the calculations of the first- and second-order moments differ in the third decimal place. As the order of the computed moments increases, to ensure the required accuracy, we must increase the power of the approximation polynomials. To compute the moments of the third and the fourth order (asymmetry and excess), integration of the investigated system of equations at no fewer than 9 points is required.

TABLE 1. Probability Characteristics of the Duration of Particle Stay as Functions of the Diffusion Coefficient

Average duration of stay m_τ, min	Diffusion coefficient $D\cdot 10^3$ m/sec			
	1	3	5	10
30	$\boxed{88.8}$ $\frac{4.5}{0.15}$	$\boxed{29.6}$ $\frac{7.6}{0.25}$	$\boxed{17.8}$ $\frac{9.8}{0.33}$	$\boxed{8.4}$ $\frac{13.4}{0.46}$
40	$\boxed{66.7}$ $\frac{6.8}{0.17}$	$\boxed{22.7}$ $\frac{11.6}{0.29}$	$\boxed{13.3}$ $\frac{14.9}{0.37}$	$\boxed{6.7}$ $\frac{20.1}{0.50}$
50	$\boxed{53.3}$ $\frac{9.6}{0.19}$	$\boxed{17.8}$ $\frac{16.3}{0.33}$	$\boxed{10.6}$ $\frac{20.6}{0.41}$	$\boxed{5.3}$ $\frac{27.6}{0.55}$
60	$\boxed{44.4}$ $\frac{12.6}{0.21}$	$\boxed{14.8}$ $\frac{21.3}{0.35}$	$\boxed{8.9}$ $\frac{24.3}{0.45}$	$\boxed{4.4}$ $\frac{35.5}{0.59}$

Note: In Table 1, the Peclet number is indicated in the rectangle, the standard deviation of the random quantity is over the bar, and the coefficient of variation is indicated under the bar.

The interpolation method is used to determine the probability characteristics of the temperature and degree of material burning in a fluidized bed.

Numerical Analysis. We investigated the probability characteristics of the duration of stay in the range of the diffusion coefficients that were obtained experimentally for different fluidized-bed treated materials [5]. Table 1 gives the results of a numerical investigation of the probability characteristics of the duration of particle stay in the reactor for different values of the parameters of the process. The standard deviation of the duration of stay and the coefficient of variation ($V = \sigma_\tau/m_\tau$) increase as the Peclet number decreases. Only the coefficient of variation is determined uniquely by the Peclet number. The variance of the duration of particle stay depends on the expectation of the duration of stay and the diffusion coefficient. The investigation of the limit of expression (2) as Pe $\to 0$ for m_τ = const, which corresponds to an unbounded increase in the diffusion coefficient, shows that $\sigma_\tau \to m_\tau$, i.e., the standard deviation is bounded by the expectation of the duration of stay. In this case, the process of particle motion is similar to the regime of ideal mixing, while the law of particle distribution by the duration of stay tends to an exponential law. As Pe $\to \infty$ the variance, asymmetry, and excess tend to zero, while the process of particle motion tends to the regime of ideal displacement.

Analysis of the results in Table 1 indicates that the standard deviation increases as the flow velocity of bed motion decreases, more rapidly than the expectation of the duration of stay. This fact is reflected in an increase in the coefficient of variation.

In the Peclet number range in question, the density of the distribution of the duration of stay is similar to a normal law in character; however, it is distinguished by asymmetry and excess. With the aim of simplifying the calculations we take the assumption of the normal distribution law.

The process of limestone dissociation that is modeled by the unknown-boundary problem is significantly nonlinear, in connection with which the expectation of the degree of burning in the general case will not be equal to the degree of burning that is calculated by mathematical model (2)-(7) for a burning time equal to the expectation. It is of practical interest to establish the limits of the standard deviation of the duration of stay within which the equality $M[\gamma(\tau)] = \gamma(m_\tau)$ can be considered approximately satisfied.

Table 2 gives the probability characteristics of the particles treated in the burning zone. As the standard deviation of the duration of stay increases, with its expectation being constant, not only the variance but also the expectations of all the characteristics of a particle change, which confirms the nonlinearity of the corresponding

TABLE 2. Probability Characteristics of the Temperatures and Degree of Limestone Particle Burring for Different Standard Deviations of the Duration of Stay in the Burning Zone

Standard deviation of the duration of stay σ_τ, min	Mean-mass particle temperature, °C	Particle surface temperature, °C	Radius of the unreacted nucleus $s \cdot 10^4$, m	Degrees of particle burning γ, %
0	902	1054	132	83.2
1	$\frac{902}{0.7}$	$\frac{1054}{0.6}$	$\frac{132}{1.6}$	$\frac{83.2}{0.6}$
5	$\frac{902}{3.6}$	$\frac{1054}{2.8}$	$\frac{133}{7.9}$	$\frac{82.9}{3.1}$
10	$\frac{902}{7.5}$	$\frac{1053}{6.6}$	$\frac{134}{16.1}$	$\frac{82.0}{6.8}$
15	$\frac{901}{11.6}$	$\frac{1051}{15.1}$	$\frac{135}{25.0}$	$\frac{80.2}{12.3}$

Note: In Tables 2 and 3, the expectation is indicated over the bar while the root-mean-square deviation of the random quantity is indicated under the bar.

TABLE 3. Probability Characteristics of the Degree of Limestone Particle Burning for Different Durations of Stay and Burning Temperatures

Duration of stay m_τ/σ_τ, min	Burning temperature, °C	Degree of burning m_γ/σ_γ, %
$\frac{30}{7.6}$	1180	$\frac{81.5}{7.0}$
	1250	$\frac{86.0}{6.5}$
$\frac{40}{11.7}$	1100	$\frac{81.5}{8.4}$
	1155	$\frac{86.0}{7.8}$
$\frac{50}{16.3}$	1100	$\frac{86.0}{9.2}$
	1250	$\frac{94.0}{7.2}$
$\frac{60}{21.3}$	1100	$\frac{89.0}{10.0}$
	1200	$\frac{94.1}{8.5}$

functional dependences. The degree of nonlinearity for different particle characteristics differs significantly. The maximum nonlinearity is offered by the degree of particle burning. When $\sigma_\tau < 5$ min the probability calculation can be replaced, with the permissible error, by a deterministic calculation of the temperatures and degree of burning. When $\sigma_\tau > 10$ min ignoring the random time of particle stay leads now to substantial errors in the numerical estimation of the degree of burning for a particle.

It is of interest to elucidate the controlling potential of the temperature of the heating medium in the burning zone. The development of a temperature regime of burning consists in determining the zone temperature that will ensure the specified degree of particle burning γ^*, i.e.,

$$m_\gamma = \gamma^* . \tag{8}$$

In terms of mathematics, the solution of the problem reduces to a one-dimensional search for a $T_{\text{h.m}}$ that will ensure the satisfaction of equality (8). To determine the required temperature, we can use the dichotomy, Newton, and other methods.

The character of the process and the structure of the object under consideration are such that they give no way of stating the problem of minimization of the variance of the degree of particle burning. In this connection, each level of degree of burning will have its own variance.

Analysis of the possibilities of controlling the degree of burning and its variance by the results of the calculations of Table 3 permits the conclusions that only the expectation can be controlled by varying the burning temperature. In this case, we are unable to attain a substantial decrease in the variance. Its decrease can be expected when burning approaches 100%. However, the dynamics of the process of limestone dissociation is such that the reaction rate drops sharply as the boundary approaches the center of the particle (calculation of the velocity of boundary motion on the surface of the particle and near the center shows a fourfold decrease). A twofold increase in the average duration of stay (from 30 to 60 min for $T_{\text{h.m}} = 1100^{\circ}\text{C}$) increases the degree of burning from 74.8 to 89.0%, while the decrease in the standard deviation is only by 1.5%. In this connection, striving for the complete burning of large fractions will require a sharp reduction in the capacity of the unit or a considerable power expenditure.

A decrease in the variance of the degree of particle burning can be attained by using more intense regimes of treatment. Thus, as the average duration of stay decreases from 50 to 30 min the variance decreases by approximately 30% for the same degree of burning. In this case, we must raise the burning temperature by 130–150°C, which can violate the limitations on the condition of fusion for the particle surface and will lead again to considerable power expenditure.

Conclusions. The numerical investigations of probability characteristics for the duration of stay, temperature, and degree of particle burning enabled us to establish a number of properties of the temperature process of fluidized-bed treatment of a material that are of fundamental importance when the capacity and the quality of the obtained material are determined.

We established a nonlinearity effect for the process of fluidized-bed treatment of particles that is due to its stochastic nature and consists in a significant decrease in the expectation of the degree of burning as the variance of the duration of particle stay increases.

Analysis of the controllability of the degree of material burning showed that only the expectation can efficiently be controlled by varying the burning temperature. In this case, we are unable to substantially decrease the variance.

NOTATION

d, coefficient of particle diffusion in bed; L, working length of reactor; u, flow-rate velocity of material motion; τ, duration of particle stay in bed; Pe, Peclet number; $T(x, t)$, material temperature at point x at instant t; λ_i, ρ_i, and c_i, thermal conductivity, density, and heat capacity of material, respectively, $i = 1$, limestone, $i = 2$, lime; α^{Σ}, total heat-transfer coefficient; T^*, temperature of the reaction of limestone dissociation; κ, specific latent heat of the reaction of limestone dissociation; $T_{\text{h.m}}$, heating-medium temperature; $1(T - T^*)$, unit Heaviside function; γ, degree of particle burning; M, m, symbols for expectation of random quantity; D, symbol for variance of random quantity; σ, standard deviation of random quantity; v_i standard Chebyshev-type nodes; r_i, Christoffel numbers that correspond to the chosen nodes v_i; n, number of different values of random quantity (power of interpolation polynomial).

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